

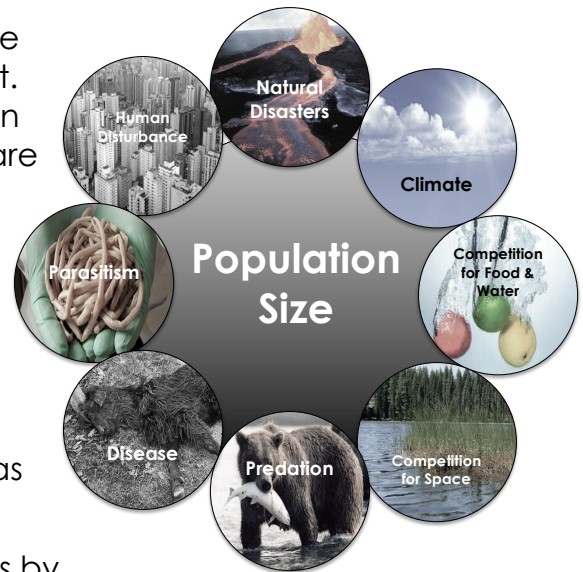
## Modeling Carrying Capacity

### HASPI Medical Biology Lab 09a

#### Background

#### Carrying Capacity and Limiting Factors

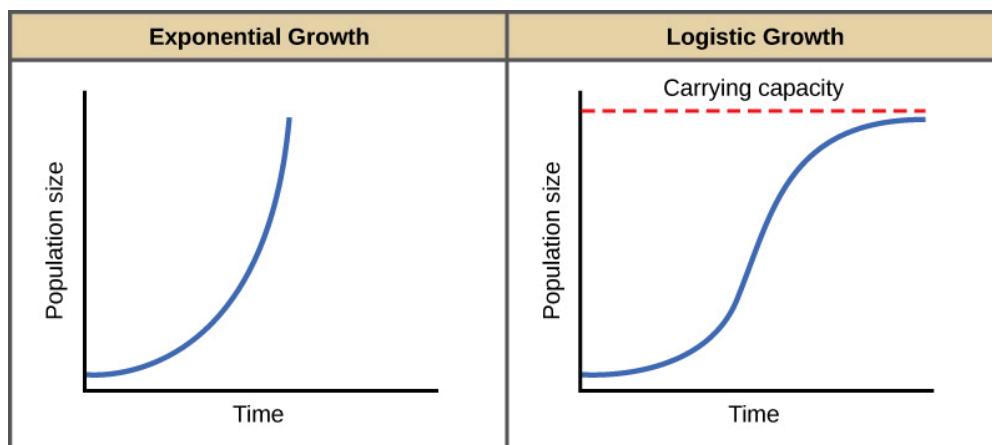
The **carrying capacity** of an ecosystem is considered the maximum population size that environment can support. The growth of a population is controlled by factors within the environment that limit the population's size. These are called **limiting factors**. Limiting factors most commonly include availability of living and nonliving resources, predation, competition, and disease. They can be considered **density dependent**, meaning the factors' impact is based on the size (density) of the population, or **density independent**, meaning the factors will have the same impact regardless of the population size. For example, competition is density dependent, whereas a natural disaster is density independent.



Carrying capacity moderates the growth of populations by slowing, stopping, or increasing growth that is dependent upon limited resources or conditions. For example, if the food source of a deer population can only support 1,000 deer, that is the carrying capacity for that population. As the population of deer increases, the food source decreases, and competition occurs. Those deer that are better adapted to obtain the food source will survive, while others will die off.

#### Exponential Growth vs. Logistic Growth

The two most common types of population growth are exponential and logistic population growth. **Exponential growth** occurs when there are unlimited natural resources available. Exponential growth cannot occur indefinitely, as eventually a population will run out of resources. Logistic growth takes into account these limiting factors and carrying capacity of a population in a specific ecosystem. In **logistic growth**, the depletion of resources will slow the rate of growth, eventually reaching a plateau. This is the carrying capacity of a population, and is represented by the letter **K**. The following graphs visually demonstrate the difference between exponential and logistic growth.



<https://figures.boundless.com/21346/full/figure-45-03-01.jpg>

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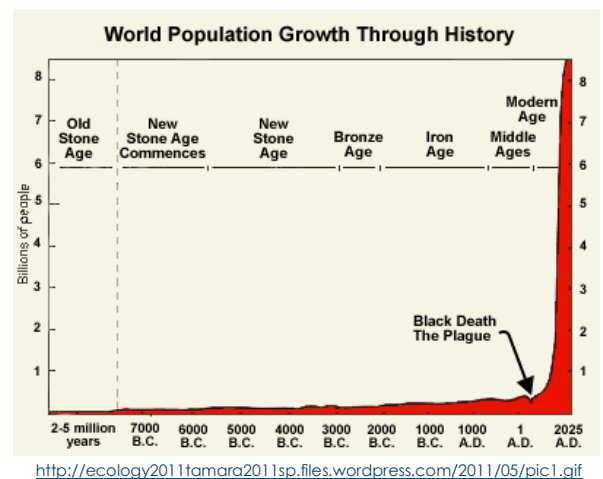
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While observations of growth in a population would produce the most accurate results, this is not always possible, particularly in species that grow very large populations quickly such as bacteria. For this reason, scientists use mathematical and computational models to represent population growth.

Exponential Growth Equation	Logistic Growth Equation
<b><math>N = N_0e^{rt}</math></b>	<b><math>dN/dt = rN [1-N/K]</math></b>
<p><b>N</b> = number of individuals at a given time  <b>N<sub>0</sub></b> = the starting population  <b>e</b> = 2.718 (constant)  <b>r</b> = growth rate (birth rate – death rate)  <b>t</b> = time interval</p>	<p><b>dN/dt</b> = change in population size  <b>r</b> = growth rate (birth rate – death rate)  <b>N</b> = number of individuals  <b>K</b> = carrying capacity</p>
<p style="text-align: center;"><b>Example</b></p> <p>If a population starts with 3,000 individuals, and has a growth rate of 0.083, calculate the population size after 6 months.</p> <p style="text-align: center;"><math>N = 3000e^{(0.083)(6)}</math>  <math>N = 4945</math></p> <p><i>This means that the population increased from 3,000 to 4,945 over a 6-month period.</i></p>	<p style="text-align: center;"><b>Example</b></p> <p>If a population has 10 individuals, a carrying capacity of 100 individuals, and a growth rate of 0.15, calculate the change in population size over a month.</p> <p style="text-align: center;"><math>dN/dt = (0.15)(10)[1-(10/100)]</math>  <math>dN/dt = (0.15)(10)(0.9)</math>  <math>dN/dt = 1.35</math></p> <p><i>This means the population changed by 1.35 individuals in a month.</i></p>

### Carrying Capacity of the Human Population

Human population growth is a bit more complex. There are additional variables, such as industrialization and healthcare, that must be considered. In general, when the population is below carrying capacity, it will increase; and when it is above carrying capacity, it will decrease. While a wide range of estimates have been proposed, the carrying capacity of the human population on Earth is theorized to be approximately 10 billion people. The world population at the end of 2013 was 7.1 billion people, and is projected to reach its carrying capacity by the year 2050.



### Review Questions – answer questions on a separate sheet of paper

1. What is carrying capacity?
2. Why is carrying capacity of populations important to a healthy ecosystem?
3. What are limiting factors? Give 3 examples.
4. What is the difference between a density dependent and a density independent limiting factor? Give an example of each.
5. Compare and contrast exponential growth vs. logistic growth.
6. Why are mathematical or computational models useful when representing population growth?
7. What is the exponential growth equation? The logistic growth equation?
8. What is the carrying capacity of the human population? When are we projected to reach that population?

Name(s):

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## Modeling Carrying Capacity

### HASPI Medical Biology Lab 09a

#### Introduction



While field research is the most accurate, it is sometimes unrealistic and time consuming to collect data on large populations and ecosystems. It is often easier and more productive to use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.



#### Materials


Calculator


Graph paper/Graphing software

#### Directions


### Part A. A Computational Model

Task	Response
Go to the following website:  <i>Connecting Concepts: Interactive Lessons in Biology</i> <i>Ecology &gt; Population Dynamics</i>  <b><a href="http://ats.doit.wisc.edu/biology/ec/pd/pd.htm">http://ats.doit.wisc.edu/biology/ec/pd/pd.htm</a></b>	
On this website, you will have the opportunity to simulate exponential and logistic population growth using a computational model.	
<b>TOPIC 1: Exponential Growth</b>	
<b>1</b> Scroll down to the bottom of the webpage to the "Lesson Topics."  <div style="text-align: center;">  <p>Click on:</p> </div>	a. Why are models useful?  b. Why are zebra mussels considered an invasive species?
<b>2</b> Read the information on each page, and answer the questions as you proceed through the website.	
<b>3</b> Click the forward arrow on the bottom right of the page to proceed through the simulation model.	c. What question will you be answering throughout the TOPIC 1 simulation?
<b>Step 1: Expert predictions</b>	
<b>4</b> Summarize Professor Barrios' prediction. Draw a diagram to support her prediction.  <div style="text-align: center;">  </div>	Prediction:

<b>5</b>	<p>Summarize Professor Nielsen's prediction. Draw a diagram to support his prediction.</p> 	<p>Prediction:</p>
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<b>6</b>	<p>Summarize Professor Cho's prediction. Draw a diagram to support his prediction.</p> 	<p>Prediction:</p>
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**Step 2: Collect data**

<b>7</b>	<p>Click the  button 3 times to collect data from sites 1, 2, and 3 for Year 1. Record the results in Table 1.</p>
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<b>8</b>	<p>Click the  button to obtain the annual average of the samples. Record the results in Table 1.</p>
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<b>9</b>	<p>Click the  button. Repeat steps 7 and 8 to obtain data for Year 2. Continue to the next year until all 5 years of data have been recorded in Table 1.</p>
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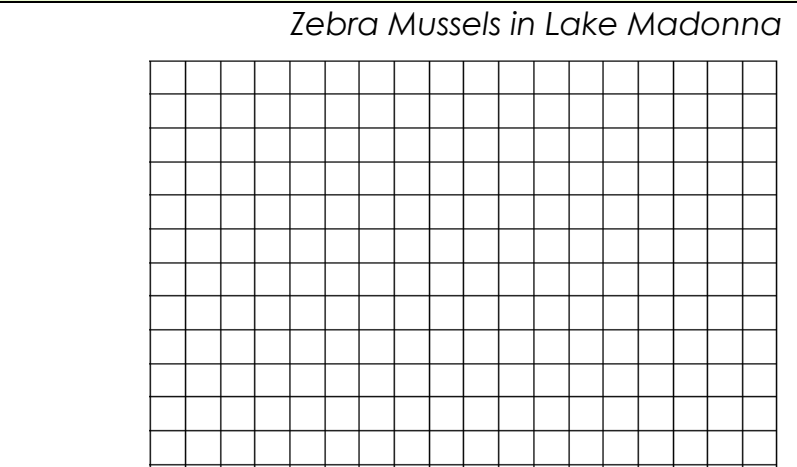
Table 1. Number of Zebra Mussels in Lake Madonna					
Site	Year 1	Year 2	Year 3	Year 4	Year 5
1					
2					
3					
Average (#ZM/m <sup>2</sup> )					

**Step 3: Construct growth curve; fit with model**

<b>10</b>	<p>Use the pull-down menus to select the variables, scales, and units for the graph. You must submit the correct answers before a graph will be created.</p>
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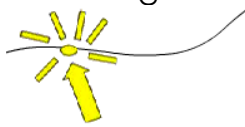
<b>11</b>	<p>Plot the data by pushing the red button for each set of data, starting with Year 1.</p>
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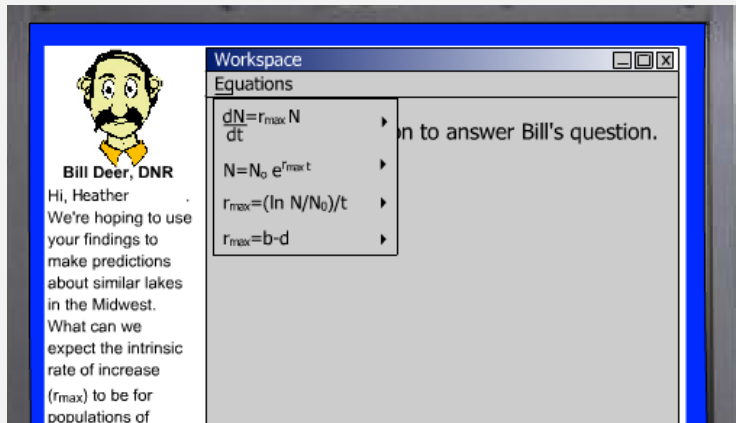
<b>12</b>	<p>Draw a line through the data at the data points, and recreate the graph at right. Do not forget to include the variables, scales, and units.</p>
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<b>13</b>	Does this graph represent exponential or logistic population growth? Explain your answer.	Answer:
<b>14</b>	What is the number of individuals added each new year?	Answer:
<b>15</b>	Which professor predicted the correct population growth for the zebra mussels?	Answer:
<b>16</b>	Which population growth equation represents the population growth of the zebra mussels (Record BOTH versions)?	Answer:

**Step 4: Present data at a teleconference**

<b>17</b>	<p style="text-align: center;">Click "<b>here</b>" to continue. Connect to the teleconference and answer each question. Use the space provided on the right side below to show your work.</p> <div style="text-align: center;">  <p>Click here to connect</p> </div>
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<b>18</b>	<p>Within the workspace, select an equation and use the equation to find a correct answer for each question. The questions are available in the simulation and below. You cannot move on to the next question until a correct answer is provided. If a question has multiple parts, provide an answer for ALL parts.</p>	
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<b>19</b>	<p><b>Bill Deer, DNR</b>  <i>We're hoping to use your findings to make predictions about similar lakes in the Midwest. What can we expect the intrinsic rate of increase (r) to be for populations of invading zebra mussels?</i></p>	Answer:
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<b>20</b>	<p><b>Desi Rent</b>  <i>I'm worried about my boat engine getting clogged by mussels and my kids cutting their feet on the shells. Can you tell me what zebra mussel densities will be next year (year 6)?</i></p>	Answer:
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21	<p><b>Dr. Anusha Gandhari, GLC</b>  <i>We've got a lake in Michigan about the same size as Lake Madonna that has 100 ZM/m<sup>2</sup>. We're wondering, based on your work, how many years do we have until the mussels reach a density of about 20,000 per m<sub>2</sub>?</i></p>	Answer:
22	<p><b>Jean Shaw, Mayor's aide</b>  <i>I see the data you've collected. There's one thing I just can't understand: why doesn't the population grow in a straight line? Can you explain to me why there is a j-shaped curve?</i></p>	Answer:
23	Click the forward arrow on the bottom right of the page to proceed to the next simulation model: logistic growth.	
<b>TOPIC 2: Logistic Growth</b>		
1	If you have not done so already, make sure you are on TOPIC 2: Logistic Growth.	
2	Read through the directions on "How to proceed" through this simulation. Proceed through the simulation and answer the questions below.	
<b>1 Density-dependence</b>		
3	How would you describe the rate of growth of the fish population? Draw a basic graph of the growth (units are not necessary in this case).	Answer:
<b>2 Carrying capacity</b>		
4	What is the main difference between the logistic and exponential equations?	Answer:
5	What does K represent?	Answer:
6	What is carrying capacity?	Answer:
7	What happens biologically as a population reaches its carrying capacity?	Answer:

Name(s):

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Date:

**3 Equation components****8**

Let's empty out the lake and divide it into 50 equally-sized squares. Each square contains enough resources to support one fish on average. What's the carrying capacity of this lake?

Answer:

**9**

How should we think about  $[1-(N/K)]$ ?

Answer:

**10**

Add fish to the lake and record how the fish population affects  $[1-(N/K)]$  in Table 2. What happens to the unused portion of the carrying capacity when the population increases?

Answer:

Table 2. Used vs. Unused Carrying Capacity	
N	$[1-(N/K)]$

**11**

What is the logistic growth equation?

Answer:

**12**

What is the difference between  $r_{\max}$  and  $r$ ?

Answer:

**13**

How do birth and death rates relate to  $r_{\max}$ ?

Answer:

**14**

As a population approaches  $K$ , what happens to  $r$ ?

Answer:

**15**

If  $K=50$  and  $r_{\max} = 0.5$ , when will  $r$  be exactly half of  $r_{\max}$ ?

Answer:

**4 Summary activity****16**

What are four properties of logistic growth?

Answer:

**17**

What are four properties of exponential growth?

Answer:

**18**

Click the forward arrow on the bottom right of the page to proceed to the next simulation model: elephant population growth.



Name(s):

Period:

Date:

**TOPIC 3: Elephant Population Growth****1** If you have not already, make sure you are on TOPIC 3: Elephant Population Growth.**2** Where is Kruger National Park, and why is it important to the elephant population?

Answer:

**3** How has the elephant population been managed? Why is this program important to Kruger National Park?

Answer:

**4** Has the growth from 1903 - 1996 been exponential or logistic? Explain your answer.

Answer:

**5** What happened to the elephant population from 1903-1920?

Answer:

**6** What happened to the elephant population from 1920-1940?

Answer:

**7** What happened to the elephant population from 1940-1960?

Answer:

**8** What happened to the elephant population from 1960-1996?

Answer:

**9** Calculate values, and complete the chart to the right as you proceed through the simulation.

Time	Year	$N^*$	$1-(N/K)$	$dN/dt$
1	1905	10		
2	1930			
	1935			
	1940			
3	1944			
	1946			
	1950			
4	1996			

\*The values of  $N$  used here are estimated from the model using  $r_{max} = .15$  and  $K=7500$ . You will look at the actual recorded data for  $N$  later in this tutorial.



Name(s):

Period:

Date:

<b>10</b>	How does the logistic model for the elephant population compare to the actual data recorded by park wardens and biologists?	Answer:
<b>11</b>	Sketch the graph comparing logistic growth and the actual data for the elephant population in Kruger National Park.	
<b>12</b>	How effective was the model in predicting the real elephant population growth? Explain your answer.	Answer:

Name(s):


Period:

Date:

**Part B. A Mathematical Model**

Task	Response
Answer the questions on the right; for the word problems use the exponential and logistic growth equations found in the Background section (first pages). Show your work.	
1 What is the difference between exponential and logistic population growth? Give the equations for each.	
2 In the exponential growth equation, what do each of the following variables represent?	$N =$ $N_0 =$ $e =$ $r =$ $t =$
3 In the logistic growth equation, what do each of the following variables represent?	$dN/dt =$ $N =$ $K =$ $r =$
4 What is carrying capacity (K)? Why do populations fluctuate around their carrying capacity?	

**Word Problem A**

Task	Response
Human population growth and density have a profound impact on health and medicine. If the human population size in 2008 was 6.8 billion, what is the projected population size in the year 2018? Assume the population is growing exponentially, and the growth rate ( $r$ ) is 0.014.	
List the values for each variable. Put the values into the exponential growth equation to obtain the answer. Show your work.	$N =$ _____ $N_0 =$ _____ $e =$ _____ $r =$ _____ $t =$ _____

Name(s):

Period:

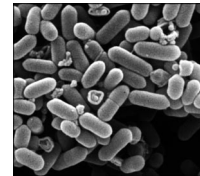
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### Word Problem B

#### Task

#### Response

A medical researcher is attempting to find the growth rate of a new strain of bacteria that has caused infections in several patients. At time 0 hours, the researcher placed 100 bacteria on an agar plate to grow. Five hours later, 600 bacteria were counted. Assuming the bacteria are growing exponentially, what is the growth rate ( $r$ ) for the bacteria?



List the values for each variable. Put the values into the exponential growth equation to obtain the answer. Show your work.

$$N = \underline{\hspace{2cm}}$$

$$N_0 = \underline{\hspace{2cm}}$$

$$e = \underline{\hspace{2cm}}$$

$$r = \underline{\hspace{2cm}}$$

$$t = \underline{\hspace{2cm}}$$

### Word Problem C

#### Task

#### Response

Round worms are parasites that can infect humans who consume food or water containing round worm eggs. Approximately 1.2 billion people globally are infected by round worms. A patient has been diagnosed with round worms that do not seem to be responding to antibiotics. Initially, the round worm population was estimated to be 2,000. It is estimated that the birth rate of the worms is 0.34 and the death rate is 0.03. If the population is growing exponentially, what is the population predicted to be after 30 days? (Remember  $r = \text{birth rate} - \text{death rate}$ )



List the values for each variable. Put the values into the exponential growth equation to obtain the answer. Show your work.

$$N = \underline{\hspace{2cm}}$$

$$N_0 = \underline{\hspace{2cm}}$$

$$e = \underline{\hspace{2cm}}$$

$$r = \underline{\hspace{2cm}}$$

$$t = \underline{\hspace{2cm}}$$

Name(s):

Period:

Date:

### Word Problem D

#### Task

#### Response

*Salmonella* is a group of bacteria that can cause food poisoning. The growth rate of *Salmonella* is 0.42. If a person consumes a population of *Salmonella* estimated to be 40 organisms, and it is assumed that *Salmonella* grow exponentially, calculate the population size at 1 hour, 2 hours, 3 hours, 4 hours, 5 hours, 6 hours, 12 hours, and 24 hours. Graph the results.



Put the values into the exponential growth equation to obtain the answer. Show your work.

Record the results on separate graph paper. Label your axes.

- Based on what you have learned about exponential growth, describe the population growth of *Salmonella* after 24 hours.
- What do you think would happen to the patient if this population continued to grow with unlimited resources? Explain your answer.
- Hypothesize how this population could be controlled in the patient.

Name(s):

Period:

Date:

## Word Problem E

## Task

## Response

*Candida albicans* is a fungus that can cause yeast infections. A population of *C. albicans* is found in an infection in a patient's mouth. This condition is known as oral thrush. The lab reported that the current population is approximately 150,000 individuals. If the carrying capacity is 800,000 individuals, and growth rate is 0.5 individuals/day, what is the change in population size for *C. albicans* each day?



List the values for each variable. Put the values into the logistic growth equation to obtain the answer. Show your work.

$$dN/dt = \underline{\hspace{2cm}}$$

$$N = \underline{\hspace{2cm}}$$

$$K = \underline{\hspace{2cm}}$$

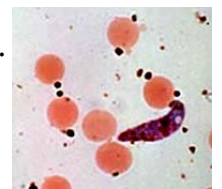
$$r = \underline{\hspace{2cm}}$$

## Word Problem F

## Task

## Response

Malaria is a disease caused by a parasite, and it produces flu-like symptoms. A severe infection can lead to death. A patient has been diagnosed with malaria. The carrying capacity of malaria is 25,000, and the population is currently 2,500. From day 1 of the infection to day 2, the population increased by 1,400. What is the growth rate of this malarial infection, and how long before the population reaches carrying capacity (resulting in the death of the patient)?



List the values for each variable. Put the values into the logistic growth equation to obtain the answer. Show your work.

$$dN/dt = \underline{\hspace{2cm}}$$

$$N = \underline{\hspace{2cm}}$$

$$K = \underline{\hspace{2cm}}$$

$$r = \underline{\hspace{2cm}}$$

Name(s):

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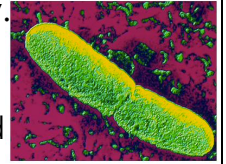
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## Word Problem G

## Task

## Response

*Yersinia pestis* is the bacteria that caused the Black Death during the 14<sup>th</sup> century. *Y. pestis* was most likely transported to Europe on a trading ship. The population of a small coastal area of Europe during this time was 5 million, and the growth rate of infection was 0.24. For this word problem we will assume that 100 infected people entered Europe from the trading ship. What was the change in the infected population per month in 1348 AD? Graph the results.



Put the values into the logistic growth equation to obtain the answer. Show your work.

Create 3 graphs on a separate sheet of paper: one graph for  $N$ , one for  $1 - (N/K)$ , and one for  $dN/dt$ . Label your axes. Answer the questions below based on your graphs.

$$r = 0.24$$

$$K = 5,000,000$$

Month 1348 AD	$N$	$1 - (N/K)$	$dN/dt$
Jan	100		
Feb	1,260		
Mar	36,504		
Apr	98,850		
May	212,675		
Jun	572,140		
Jul	1,000,250		
Aug	1,765,743		
Sep	3,000,140		
Oct	3,975,001		
Nov	4,300,500		
Dec	4,850,025		

- What happens to the infected population ( $N$ ) as it reaches carrying capacity ( $K$ )?
- What happens to the unused carrying capacity [ $1 - (N/K)$ ] as the infected population increases?
- Summarize how the infected population changed per month.